Efficient Representation of Timed UML 2 Interactions

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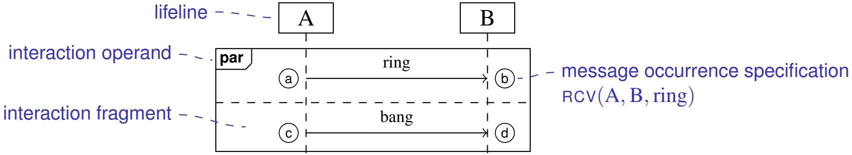
Traces (of Untimed Interactions)



- Direct enumeration of traces of occurrence specifications
  - 6 traces

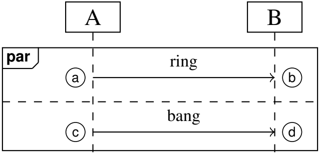
a b c d   c a b d

a c b d   c a d b

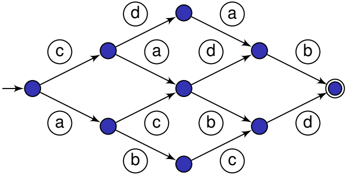
a c d b   c d a b

[St. 2003]

Phase Automata

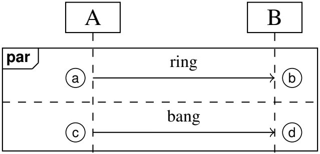


- Traces captured by runs from initial to final state
  - 9 states, 12 transitions

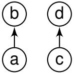


[K., Wuttke 2006]

Prime Event Structures (1)

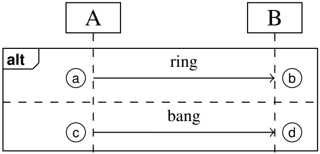


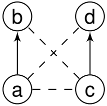
- Traces captured by linearisations of maximal configurations
  - Events: $E = \{ⓐ, ⓑ, ⓒ, ⓓ\}$
    - Set of events (occurrence specifications), which may occur
  - Causality relation on events: ⓐ $\prec$ ⓑ, ⓒ $\prec$ ⓓ
    - Partial order, describing which event must occur before which other
  - Conflict relation w.r.t. $\preceq$: $\sharp = \emptyset$
    - Symmetric exclusion relation with: if $e \sharp e' \preceq e''$, then $e \sharp e''$
  - Maximal configuration
    - maximal downwards closed set of events not containing conflicting events



[Küster-Filipe 2006]

Prime Event Structures (2)





- Events: $E = \{$ⓐ, ⓑ, ⓒ, ⓓ$\}$
- Causality relation: ⓐ $\prec$ ⓑ, ⓒ $\prec$ ⓓ
- Conflict relation w.r.t. $\leq$: ⓐ $\sharp$ ⓒ
- Maximal configurations

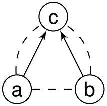
Prime Event Structures (3)

Consider

- strict(alt(ⓐ, ⓑ), ⓒ)

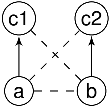
For prime event structure representation

- ⓐ and ⓑ are in conflict
- ⓐ and ⓑ are before ⓒ



Leads to duplication of ⓒ into (c1) and (c2)

- ⓐ and ⓑ are in conflict
- ⓐ before (c1), ⓑ before (c2)



In strict(alt(ⓐ, ⓑ), T) all of T has to be duplicated.

Also problems with expressing asymmetric conflicts, like for break.

Approach

Keep constraints approach of event structures

- direct representation of basic interactions
  - partially ordered occurrence specifications
- compact format for par, strict, seq

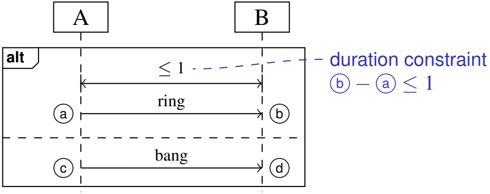
Avoid necessity of duplication and symmetric conflicts

- asymmetric event structures, flow event structures

Integrate timing constraints

- duration constraints $o_2 - o_1 \bowtie d$ with $\bowtie \in \{<, \leq, \geq, >\}$
- duration constraints $\ell \bowtie d$ for an interaction fragment

Example



- Interaction structure (O, R, X, Θ)

$$O = \{ⓐ, ⓑ, ⓒ, ⓓ\}$$

$$R = \{ⓐ \to ⓑ, ⓒ \to ⓓ\}$$

$$X = \{ⓐ \rightsquigarrow ⓒ, ⓐ \rightsquigarrow ⓓ, ⓒ \rightsquigarrow ⓐ, ⓓ \rightsquigarrow ⓐ\}$$

$$\Theta = ⓑ - ⓐ \leq 1$$

- Traces of (O, R, X, Θ)

$$\langle \mathsf{SND}(\mathrm{A}, \mathrm{B}, \mathrm{ring}), t_1\rangle \langle \mathsf{RCV}(\mathrm{A}, \mathrm{B}, \mathrm{ring}), t_2\rangle, \quad t_1, t_2 \in \mathbb{R}_{\geq 0}, \quad t_2 - t_1 \leq 1$$

$$\langle \mathsf{SND}(\mathrm{A}, \mathrm{B}, \mathrm{bang}), t_3\rangle \langle \mathsf{RCV}(\mathrm{A}, \mathrm{B}, \mathrm{bang}), t_4\rangle, \quad t_1, t_2 \in \mathbb{R}_{\geq 0}$$

Interaction Structures

- Finite set of occurrence specifications O
  - events conforming to these occurrence specifications are allowed to be observed
- Binary relation $R \subseteq O \times O$ specifying a causality relation
  - partial order $\preceq_R$
  - event ordering on a trace must not contradict $\preceq_R$
- Binary relation $X \subseteq O \times O$ specifying an inhibition relation w.r.t. R
  - irreflexive relation $\rhd_{(R,X)}$ with $o_2 \rhd_{(R,X)} o_3$ iff there is an $o_1 \in O$ with $o_1 \preceq_R o_2$ and $(o_1, o_3) \in X$
  - for a trace, $o_1 \rhd_{(R,X)} o_2$ inhibits an event conforming to o_2 to occur after an event conforming to o_1
- Timing constraint Θ
  - conjunctive or disjunctive combination of timing constraints of the form $o_2 - o_1 \bowtie d$

Traces of Interaction Structures

Interaction structure (O, R, X, Θ)

Sequence of occurrence specifications $o_1 \ \ldots \ o_k$ conforms to R and X if o_j minimal element of $(O_j, \preceq_R \cap (O_j \times O_j))$ with

$$O_j = O \setminus (\{o_1, \ldots, o_{j-1}\} \cup \{o \in O \mid \exists 1 \leq i \leq j-1 \,.\, o_i \rhd_{(R,X)} o\})$$

- each o_j minimal w.r.t. causality, and not inhibited

$o_1 \ \ldots \ o_k$ allowed by (O, R, X, Θ) if it conforms to R and X and it is maximal

- no $o \in O \setminus \{o_1, \ldots, o_k\}$ such that $o_1 \ \ldots \ o_k \, o$ conforms to R and X

Trace of timed events $e_1 \ \ldots \ e_k$ for (O, R, X, Θ) if

- there is a trace $o_1 \ \ldots \ o_l$ allowed by (O, R, X, Θ)
- the events e_i can be correctly labelled with occurrence specifications o_j
- the timing constraint Θ is satisfied w.r.t. to the labelling

Examples (1)

Interaction structure (O, R, X, Θ) for strict(alt(ⓐ, ⓑ), ⓒ)

$$O = \{ⓐ, ⓑ, ⓒ\}$$

$$R = \{ⓐ \rightarrow ⓒ, ⓑ \rightarrow ⓒ\}$$

$$X = \{ⓐ \rightsquigarrow ⓑ, ⓑ \rightsquigarrow ⓐ\}$$

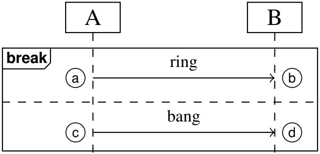
$$\Theta = \text{true}$$

Sequences of occurrence specifications allowed by (O, R, X, Θ)

ⓐⓒ

ⓑⓒ

Examples (2)



- Interaction structure (O, R, X, Θ)

$$O = \{\textcircled{a}, \textcircled{b}, \textcircled{c}, \textcircled{d}\}$$

$$R = \{\textcircled{a} \rightarrow \textcircled{b}, \textcircled{c} \rightarrow \textcircled{d}\}$$

$$X = \{\textcircled{c} \leadsto \textcircled{a}, \textcircled{c} \leadsto \textcircled{b}, \textcircled{d} \leadsto \textcircled{a}, \textcircled{d} \leadsto \textcircled{b}, \textcircled{b} \leadsto \textcircled{c}, \textcircled{b} \leadsto \textcircled{d}\}$$

$$\Theta = \mathrm{true}$$

- Sequences of occurrence specifications allowed by (O, R, X, Θ)

$\textcircled{a}\textcircled{b}$

$\textcircled{a}\textcircled{c}\textcircled{d}$

$\textcircled{c}\textcircled{d}$

Deriving Interaction Structures (1)

- Basic interactions

  $$\mathcal{S}[\![(O, \rightarrow)]\!] = (O, \rightarrow, \emptyset, \mathrm{true})$$

- Strict sequencing of $\mathcal{S}[\![T_i]\!] = (O_i, R_i, X_i, \Theta_i)$

  $$\begin{aligned}\mathcal{S}[\![\mathsf{strict}(T_1, T_2)]\!] = (&O_1 \cup O_2,\\ &R_1 \cup R_2 \cup \{o_1 \rightarrow o_2 \mid o_1 \in O_1,\ o_2 \in O_2\},\\ &X_1 \cup X_2,\\ &\Theta_1 \wedge \Theta_2)\end{aligned}$$

- Weak sequencing of $\mathcal{S}[\![T_i]\!] = (O_i, R_i, X_i, \Theta_i)$

  $$\begin{aligned}\mathcal{S}[\![\mathsf{seq}(T_1, T_2)]\!] = (&O_1 \cup O_2,\\ &R_1 \cup R_2 \cup \{o_1 \rightarrow o_2 \mid o_1 \in O_1,\ o_2 \in O_2,\ o_1 \between o_2\},\\ &X_1 \cup X_2,\\ &\Theta_1 \wedge \Theta_2)\end{aligned}$$

  - where $o_1 \between o_2$ holds if o_1 and o_2 are on the same lifeline

Deriving Interaction Structures (2)

- Parallel composition of $\mathcal{S}[\![T_i]\!] = (O_i, R_i, X_i, \Theta_i)$

  $$\mathcal{S}[\![\mathsf{par}(T_1, T_2)]\!] = (O_1 \cup O_2, R_1 \cup R_2, X_1 \cup X_2, \Theta_1 \wedge \Theta_2)$$

- Alternative composition of $\mathcal{S}[\![T_i]\!] = (O_i, R_i, X_i, \Theta_i)$

  $$\begin{aligned}\mathcal{S}[\![\mathsf{alt}(T_1, T_2)]\!] = (&O_1 \cup O_2,\\ &R_1 \cup R_2,\\ &X_1 \cup X_2 \cup \{o_1 \rightsquigarrow o_2 \mid o_1 \in O_1,\ o_2 \in O_2\} \cup\\ &\qquad\qquad \{o_2 \rightsquigarrow o_1 \mid o_1 \in O_1,\ o_2 \in O_2\},\\ &\Theta_1 \wedge \Theta_2)\end{aligned}$$

- Breaking of $\mathcal{S}[\![T_1]\!] = (O_1, R_1, X_1, \Theta_1)$ by $\mathcal{S}[\![T_2]\!] = (O_2, R_2, X_2, \Theta_2)$

  $$\begin{aligned}\mathcal{S}[\![\mathsf{break}(T_1, T_2)]\!] = (&O_1 \cup O_2,\\ &R_1 \cup R_2,\\ &X_1 \cup X_2 \cup \{o_2 \rightsquigarrow o_1 \mid o_1 \in O_1,\ o_2 \in O_2\} \cup\\ &\qquad\qquad \{o_1 \rightsquigarrow o_2 \mid o_1 \in \mathrm{Max}(O_1, \preceq_{R_1}),\ o_2 \in O_2\},\\ &\Theta_1 \wedge \Theta_2)\end{aligned}$$

Deriving Interaction Structures (3)

- Timing constraints for $\mathcal{S}[\![T]\!] = (O, R, X, \Theta)$

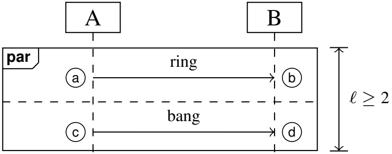
  $$\mathcal{S}[\![\mathsf{tmconstr}(T, \Gamma)]\!] = (O, R, X, \Theta \wedge \Theta_\Gamma)$$

  with expansion of $\ell \bowtie d$ for $\bowtie \in \{<, \leq\}$ into

  $$\bigwedge\{o_2 - o_1 \bowtie d \mid o_2 \in \mathrm{Max}(O, \preceq_R), o_1 \in \mathrm{Min}(O, \preceq)\}$$

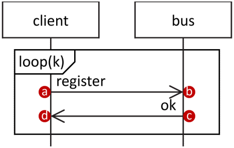
  and of $\ell \bowtie d$ for $\bowtie \in \{>, \geq\}$ into

  $$\bigvee\{o_2 - o_1 \bowtie d \mid o_2 \in \mathrm{Max}(O, \preceq_R), o_1 \in \mathrm{Min}(O, \preceq)\}$$



$$(ⓑ - ⓐ \geq 2) \vee (ⓑ - ⓒ \geq 2) \vee (ⓓ - ⓐ \geq 2) \vee (ⓓ - ⓒ \geq 2)$$

Performance



OccurrenceSpecifications

- a: snd(client, bus, register)
- b: rcv(client, bus, register)
- c: snd(bus, client, ok)
- d: rcv(bus, client, ok)

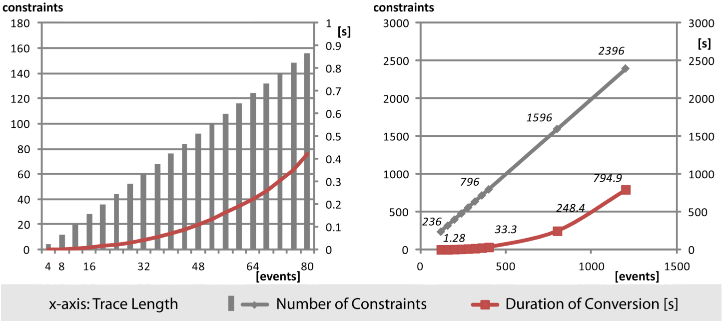
Traces

k=1: $a_1.b_1.c_1.d_1$

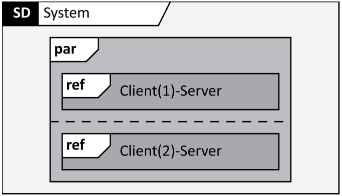
k=2: $a_1.b_1.c_1.d_1.a_2.b_2.c_2.d_2$

k=3: $a_1.b_1.c_1.d_1.a_2.b_2.c_2.d_2.a_3.b_3.c_3.d_3$

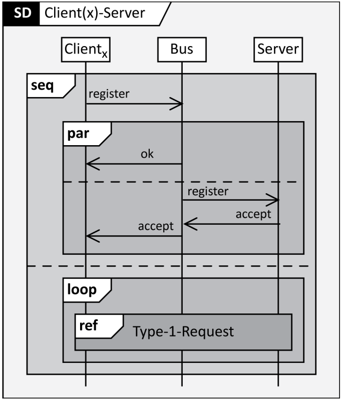
...



Conformance Checking




```
<snd(Client1, Bus,     register), 0.000>
<rcv(Client1, Bus,     register), 0.002>
<snd(Bus,     Client1, ok      ), 0.013>
<rcv(Bus,     Client1, ok      ), 0.015>
<snd(Bus,     Server,  register), 0.019>
<rcv(Bus,     Server,  register), 0.023>
<snd(Server,  Bus,     accept  ), 0.029>
<rcv(Server,  Bus,     accept  ), 0.033>
<snd(Bus,     Client1, accept  ), 0.054>
<rcv(Bus,     Client1, accept  ), 0.056>
<snd(Client1, Bus,     request ), 0.081>
<rcv(Client1, Bus,     request ), 0.083>
<snd(Bus,     Server,  request ), 0.087>
<rcv(Bus,     Server,  request ), 0.090>
<snd(Server,  Bus,     reply   ), 0.103>
<rcv(Server,  Bus,     reply   ), 0.107>
<snd(Bus,     Client1, reply   ), 0.112>
<rcv(Bus,     Client1, reply   ), 0.115>
```



Conclusions and Future Work

- Efficient representation of UML 2 interactions
  - based on asymmetric event structures
  - declarative format using constraints
- Handling of empty traces, like for opt
  - “virtual” occurrence specifications for start and end of an interaction fragment
- Inclusion of negative behaviour, i.e., neg and assert
  - In fact, a trace violating a timing constraint is negative (invalid).
- Run-time verification